

NAG Toolbox for MATLAB

f08kd

1 Purpose

f08kd computes the singular value decomposition (SVD) of a real m by n matrix A , optionally computing the left and/or right singular vectors, by using divide-and-conquer method.

2 Syntax

```
[a, s, u, vt, info] = f08kd(jobz, a, 'm', m, 'n', n)
```

3 Description

The SVD is written as

$$A = U\Sigma V^T,$$

where Σ is an m by n matrix which is zero except for its $\min(m, n)$ diagonal elements, U is an m by m orthogonal matrix, and V is an n by n orthogonal matrix. The diagonal elements of Σ are the singular values of A ; they are real and nonnegative, and are returned in descending order. The first $\min(m, n)$ columns of U and V are the left and right singular vectors of A .

Note that the function returns V^T , not V .

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: <http://www.netlib.org/lapack/lug>

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **jobz** – string

Specifies options for computing all or part of the matrix U .

jobz = 'A'

All m columns of U and all n rows of V^T are returned in the arrays **u** and **vt**.

jobz = 'S'

The first $\min(m, n)$ columns of U and the first $\min(m, n)$ rows of V^T are returned in the arrays **u** and **vt**.

jobz = 'O'

If $m \geq n$, the first n columns of U are overwritten on the array **a** and all rows of V^T are returned in the array **vt**. Otherwise, all columns of U are returned in the array **u** and the first m rows of V^T are overwritten in the array **vt**.

jobz = 'N'

No columns of U or rows of V^T are computed.

Constraint: **jobz** = 'A', 'S', 'O' or 'N'.

2: **a(lda,*) – double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The m by n matrix A .

5.2 Optional Input Parameters1: **m – int32 scalar**

Default: The first dimension of the array **a**.

m , the number of rows of the matrix A .

Constraint: $\mathbf{m} \geq 0$.

2: **n – int32 scalar**

Default: The second dimension of the array **a**.

n , the number of columns of the matrix A .

Constraint: $\mathbf{n} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, ldu, ldvt, work, lwork, iwork

5.4 Output Parameters1: **a(lda,*) – double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

If **jobz** = 'O', **a** is overwritten with the first n columns of U (the left singular vectors, stored column-wise) if $\mathbf{m} \geq \mathbf{n}$; **a** is overwritten with the first m rows of V^T (the right singular vectors, stored row-wise) otherwise.

If **jobz** \neq 'O', the contents of **a** are destroyed.

2: **s(*) – double array**

Note: the dimension of the array **s** must be at least $\max(1, \min(\mathbf{m}, \mathbf{n}))$.

The singular values of A , sorted so that $\mathbf{s}(i) \geq \mathbf{s}(i+1)$.

3: **u(ldu,*) – double array**

The first dimension, **ldu**, of the array **u** must satisfy

if **jobz** = 'S' or 'A' or **jobz** = 'O' and $\mathbf{m} < \mathbf{n}$, $\mathbf{ldu} \geq \max(1, \mathbf{m})$;
 $\mathbf{ldu} \geq 1$ otherwise.

The second dimension of the array must be at least $\max(1, ucol)$, where $ucol$ is the number of columns of U requested

$ucol = \mathbf{m}$ if **jobz** = 'A' or **jobz** = 'O' and $\mathbf{m} < \mathbf{n}$; $ucol = \min(\mathbf{m}, \mathbf{n})$ if **jobz** = 'S'.

If **jobz** = 'A' or **jobz** = 'O' and $\mathbf{m} < \mathbf{n}$, **u** contains the m by m orthogonal matrix U .

If **jobz** = 'S', **u** contains the first $\min(m, n)$ columns of U (the left singular vectors, stored column-wise).

If **jobz** = 'O' and $\mathbf{m} \geq \mathbf{n}$, or **jobz** = 'N', **u** is not referenced.

4: **vt(ldvt,*) – double array**

The first dimension, **ldvt**, of the array **vt** must satisfy

if **jobz** = 'A' or **jobz** = 'O' and $\mathbf{m} \geq \mathbf{n}$, $\mathbf{ldvt} \geq \max(1, \mathbf{n})$;
 if **jobz** = 'S', $\mathbf{ldvt} \geq \max(1, \min(\mathbf{m}, \mathbf{n}))$;
 $\mathbf{ldvt} \geq 1$ otherwise.

The second dimension of the array must be at least $\max(1, \mathbf{n})$

If **jobz** = 'A' or **jobz** = 'O' and $\mathbf{m} \geq \mathbf{n}$, **vt** contains the n by n orthogonal matrix V^T .

If **jobz** = 'S', **vt** contains the first $\min(m, n)$ rows of V^T (the right singular vectors, stored row-wise).

If **jobz** = 'O' and $\mathbf{m} < \mathbf{n}$, or **jobz** = 'N', **vt** is not referenced.

5: **info – int32 scalar**

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **jobz**, 2: **m**, 3: **n**, 4: **a**, 5: **lda**, 6: **s**, 7: **u**, 8: **ldu**, 9: **vt**, 10: **ldvt**, 11: **work**, 12: **lwork**, 13: **iwork**,
 14: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

f08kd did not converge, the updating process failed.

7 Accuracy

The computed singular value decomposition is nearly the exact singular value decomposition for a nearby matrix $(A + E)$, where

$$\|E\|_2 = O(\epsilon)\|A\|_2,$$

and ϵ is the *machine precision*. In addition, the computed singular vectors are nearly orthogonal to working precision. See Section 4.9 of Anderson *et al.* 1999 for further details.

8 Further Comments

The total number of floating-point operations is approximately proportional to mn^2 when $m > n$ and m^2n otherwise.

The singular values are returned in descending order.

The complex analogue of this function is f08kp.

9 Example

```
jobz = 'Overwrite A by tranpose(V)';
a = [0, 0.28, -0.48, 1.07, -2.35, 0.62;
     0, -1.67, -3.09, 1.22, 2.93, -7.39;
```

```

0, 0.9399999999999999, 0.99, 0.79, -1.45, 1.03;
0, -0.78, -0.21, 0.63, 2.3, -2.57];
[aOut, s, u, vt, info] = f08kd(jobz, a)

```

```

aOut =
   -0.0000   -0.2085   -0.3119    0.1069    0.4215   -0.8186
         0    0.0952   -0.3347    0.4707   -0.7760   -0.2349
   -0.0000    0.2686    0.6933    0.6265    0.1643   -0.1662
   -0.0000    0.7233    0.1391   -0.5236   -0.1698   -0.3930

s =
   9.6278
   2.8739
   1.3350
   0.4918

u =
   -0.1342    0.8243   -0.0572   -0.5470
    0.9064    0.3172   -0.0878    0.2647
   -0.1947    0.3526    0.7674    0.4989
    0.3499   -0.3092    0.6326   -0.6179

vt =
    0    0    0    0    0    0

info =
    0

```